Geombinatorics XXX(3), January 2021, 152–162

We have nothing to lose except everything[1]

Alexander Soifer, USA
asoifer@uccs.edu

Springer editors came up with a novel idea of publishing a book about lives of mathematicians at the time of pandemic. This will result in a book "Math in the Time of Corona," A. Wonders (ed.), Springer Nature, Switzerland. A shorter version of this essay will appear in this book as Chapter 27: *Thoughts at the Time of an Epidemic.* I wrote this essay on October 18, 2020 and choose to keep its immediacy and not to update it by new developments of the last two months.

March 7, 2020. I arrive in Deerfield Beach, Florida, for an International Conference on Combinatorics, Graph Theory and Computing. Wyndham Hotel's general manager, Claude Dubois, a handsome Frenchman, and I share a hug. I am a regular here.

March 10, 2020. At the conference, I do not see a single mask. All seems typical as in the years past, although Elizabeth Loew of Springer does not arrive due to the declared state of emergency in Florida and general concerns about the virus. My university Chancellor Venkat Reddy sends faculty a message ordering all courses to move online in early April, after the Spring Break.

March 14, 2020. As I am leaving for the airport, Claude Dubois and I greet each other with touching elbows. One of my students informs me that he moves his family away from the city of Colorado Springs to a remote little town due to his fear of a panic in Colorado Springs. I find it hard to believe that my city will panic – until the day after.

March 15, 2020. Colorado Springs, I am in a supermarket. Empty shelves everywhere, carts full of whatever is still available, hours-long lines to the cashiers. People look tense, ready to explode. My university Chancellor orders all courses to move online overnight. A student asks me, "How are you going to teach online if you've never done that?" – "carefully" I reply. In truth, I've never tried online instruction, but I am promised a 30-min intro to Microsoft Teams by our IT specialist, Jackie.

[1] Albert Camus, October 1957.

March 18, 2020. And so I dive into online teaching mathematics and European cinema. It is not as hard as I feared because during the first two months of the semester my students and I developed a bond, a unity of purpose. Our sessions in Microsoft Teams are reminiscent of visiting friends over Skype.

April 11, 2020. The first personal for me and professional for all tragedy – I lose a coauthor and friend of my Princeton years, the most ingenious John Horton Conway. A few lines of my remembrance have just been published [1].

July 6, 2020. Another tragedy, Ron Graham succumbs to a fatal hereditary disease. I feel like an orphan in Ramsey Theory, as our Captain leaves the ship. My tribute is now out [2].

July 12–19, 2020. China is to host the 14th International Congress on Mathematical Education (ICME-14) in Shanghai. I am invited to give a keynote address in the Topic Study Group **"**Mathematical competitions and other challenging activities." Professional duty outweighs fears of going to the birthplace of COVID-19, but a U.S. announcement of a 14-day mandatory quarantine for everyone coming back from China causes me to pause. Then the U.S. Department of State issues "China - Level 4 [highest] warning, "Do not Travel." Emphasis in bold is State Department's:

> Do not go to the People's Republic of China (PRC), including the Hong Kong Special Administrative Region (SAR), due to **COVID-19 and arbitrary enforcement of local laws.**
>
> **Country Summary:**
>
> The PRC government arbitrarily enforces local laws, including by carrying out arbitrary and wrongful detentions and through the use of exit bans on U.S. citizens and citizens of other countries without due process of law. The PRC government uses arbitrary detention and exit bans:
>
> - to compel individuals to participate in PRC government investigations,
> - to pressure family members to return to the PRC from abroad,
> - to influence PRC authorities to resolve civil disputes in favor of PRC citizens, and
> - to gain bargaining leverage over foreign governments.

> In most cases, U.S. citizens only become aware of an exit ban when they attempt to depart the PRC, and there is no reliable mechanism or legal process to find out how long the ban might continue or to contest it in a court of law.
>
> U.S. citizens traveling or residing in the PRC or Hong Kong, may be detained without access to U.S. consular services or information about their alleged crime. U.S. citizens may be subjected to prolonged interrogations and extended detention without due process of law.
>
> Security personnel may detain and/or deport U.S. citizens for sending private electronic messages critical of the PRC government.

No keynote address is worth free food and lodging in a Chinese prison. I cancel my fully arranged trip to China. Soon after the Congress is postponed by one year. ICME-14 Chair Professor Wang proudly declares that China has conquered COVID-19 while America and others (Europe) failed to stop the spread. Let me precisely quote his August 13, 2020 statement, explicitly promoting Chinese government's deception:

> China's opening-up policy remains unchanged. It is entirely due to COVID-19 that China is now taking the necessary restrictions or quarantines on its citizens and international travelers from pandemic countries and regions, and the restrictions on the routes and flight frequency of international passenger air travel. Once the global pandemic alert is lifted, the Chinese government will not restrict the normal entry and exit of foreigners for any reason or in any form. As we all know, the US State Department's political accusation in travel advisory for China is groundless and ridiculous. It can be interpreted as a political manipulation by a small group of people for a certain purpose, and it should be completely ignored.
>
> I hope my explanation will clear up your confusion.
>
> With best regards,
> Jianpan Wang
> ICME-14 chair

Our "confusion"? Detention of innocent American, Canadian, Australian, and other nationals when they attempt to *depart* [!] from China "should be

completely ignored"? After that statement, I do not think I'll go to China in July-2021.

Some amusing invitations came as well:

- "It is our great pleasure and privilege to welcome you to join the World Gene Convention, which will take place in Macao. On behalf of the Organizing Committee, we would be honored to invite you to be a chair/speaker in Module 1: Breakthroughs in Gene while presenting about *E15: From Squares in a Square to Clones in Convex Figures*."
- "It is our great pleasure and privilege to welcome you to join the Annual World Congress of Food and Nutrition, which will be held in Singapore. On behalf of the Organizing Committee, we would be honored to invite you to be a chair/speaker at Session 405: Foodborne Diseases, Carcinogenic Food while presenting about *E23: More about Love and Death* at the upcoming WCFN."

In both cases the organizers somehow dug out the titles of my "Further Explorations" from *The Colorado Mathematical Olympiad* books (Springer, New York, 2011 and 2017) and interpreted the Olympiad problem titles literally!

Permit me to give you a test. In the following paragraph, I will use Mr. X instead of the person's name. Your task is to identify who pronounced these lines, when, and who Mr. X was. Please, no cheating by googling the text:

> We will not walk in fear, one of another. We will not be driven by fear into an age of unreason, if we dig deep in our history and our doctrine, and remember that we are not descended from fearful men – not from men who feared to write, to speak, to associate and to defend causes that were, for the moment, unpopular.
>
> This is no time for men who oppose Mr. X's methods to keep silent, or for those who approve. We can deny our heritage and our history, but we cannot escape responsibility for the result. There is no way for a citizen of a republic to abdicate his responsibilities. As a nation we have come into our full inheritance at a tender age. We proclaim ourselves, as indeed we are, the defenders of freedom, wherever it continues to exist in the world, but we cannot defend freedom abroad by deserting it at home.

> The actions of Mr. X have caused alarm and dismay amongst our allies abroad, and given considerable comfort to our enemies. And whose fault is that? Not really his. He didn't create this situation of fear; he merely exploited it – and rather successfully. Cassius was right. "The fault, dear Brutus, is not in our stars, but in ourselves."

Who is Mr. X? Mussolini? Hitler? Stalin? Putin? Trump? All fit, don't they? The quote comes from March 9, 1954, CBS-TV special program by courageous Edward R. Murrow entitled "A Report on Senator Joseph R. McCarthy." Some things. like viruses, may mutate but fundamentally never change. It is hard – if even fair – to criticize foreign lands when at home the great American democracy is under siege by our own President and his loyalists. "Freedom and liberation are an unending task," warns us Italian writer Umberto Eco in his 1995 essay:

> Ur-Fascism [Eternal Fascism] is still around us, sometimes in plainclothes. It would be so much easier for us if there appeared on the world scene somebody saying, "I want to reopen Auschwitz, I want the Blackshirts to parade again in the Italian squares." Life is not that simple. Ur-Fascism can come back under the most innocent of disguises. Our duty is to uncover it and to point our finger at any of its new instances – every day, in every part of the world. Franklin Roosevelt's words of November 4, 1938, are worth recalling: "If American democracy ceases to move forward as a living force, seeking day and night by peaceful means to better the lot of our citizens, fascism will grow in strength in our land." Freedom and liberation are an unending task.

An unending task indeed. United States of Today is Exhibit A of a fragility of democracy and freedoms. In my 42 years of American life, I perceived America as a country of high courtesy – not now. What happened? Why so many support President Trump so passionately? Racism, misogynism, anti-Semitism, anti-immigrantism existed before, but it was considered in poor taste to express these prejudices in public. Folks who held such views, limited them to a private circle, and may have even felt badly about themselves. President Trump expressed these discriminatory views publicly, and by doing so freed these people from feeling of guilt. Liberated bigots will never forget their liberator, and will always support Trump, no matter what he says or does. There are also supporters, who are generally good persons attracted by profit to support Mr. Trump. I

often hear from deeply religious people that they "hate the sin but love the sinner." In the Trump years, they reversed this noble religious principle: They detest Mr. Trump as a person but love the fruits of his actions, such as packing federal courts with judges, whose rulings may be guided by their religious views. Others are infatuated by Trump's cutting taxes on big corporations and the rich at the expense of the middle class (I can provide particulars of The Tax Cuts and Jobs Act of 2017 (TCJA) supporting my conclusion).

President Trump takes "Enemy of the People" label addressing the Press straight from Stalin's criminal playbook, where this label was followed by a firing squad. Trump stirred American waters, and mud rose to the surface. President tweets "Liberate Michigan," and his Mud plots kidnapping followed by terrorist style "trial" and execution of the amazing and courageous Michigan Governor Gretchen Whitmer. Even FBI's arrest of ca. thirteen co-conspirators contemplating this kidnapping and murder did not stop Mr. Trump from continuing attacks of Governor Whitmer in his rallies and approving chants "Lock her up." Lock her up exactly for what, may I ask? For saving lives at the time of pandemic? Mud is waiting for the dog whistle to wage a civil war if Mr. Trump loses the election and calls on his "stand-by" Mud to "save and protect their beloved President." This could happen after November-3, 2020, election [I write these lines on October-18, 2020]. My hope rests with the American military, who pledged the allegiance not to the President (as was in Nazi Germany starting in late 1933) but to the Constitution. In August-2020 letter, Chairman of the Joint Chiefs of Staff Gen. Mark Milley assured members of Congress that the military will not play a role in the November election and won't help to settle any disputes if the results are contested:

> The Constitution and laws of the US and the states establish procedures for carrying out elections, and for resolving disputes over the outcome of elections ... I do not see the US military as part of this process.

President Trump nominated three Supreme Court Justices, and believes they are in his pocket. He counts on *quid pro quo* of Neil Gorsuch, Brett Kavanaugh, and Amy Coney Barrett. Futile hope, Mr. Trump: these and other federal justices are not for sale.

Mr. Trump will likely lose reelection [he did, as we learned in November], but the genie of racism, white supremacism, misogynism, anti-Semitism, anti-immigrantism is out of the bottle. It will take decades to put this ugly genie back.

The COVID-19 epidemic forced me to postpone the ready-to-run 37th Soifer (formerly Colorado) Mathematical Olympiad from April-2020 to April-2021 [now we consider a further postponement to October 2021]. Yet, an Olympiad train of thought took me to a research space. In 2010, I created – and in 2011 used the following problem in the Colorado Mathematical Olympiad (CMO).

Won't you be my neighbor! (*A. Soifer*, 2010, problem 5 of the 28th CMO in 2011). Each unit square of a 2011×2011 square grid is colored in one of 2011 colors so that each color is used. A pair of distinct colors is called a *neighbor pair* if they appear as colors of a pair of unit squares sharing a side.

(A). Find the maximum M of the number of neighbor pairs.

(B). Find the minimum m of the number of neighbor pairs.

Solution. Form a graph G with 2011 vertices, one per color, and two vertices adjacent if and only if the corresponding colors form a neighbor pair somewhere on the colored grid. The graph G is connected, for there is always a rook's path on the grid connecting unit squares of any pair of colors (Fig. 1). .

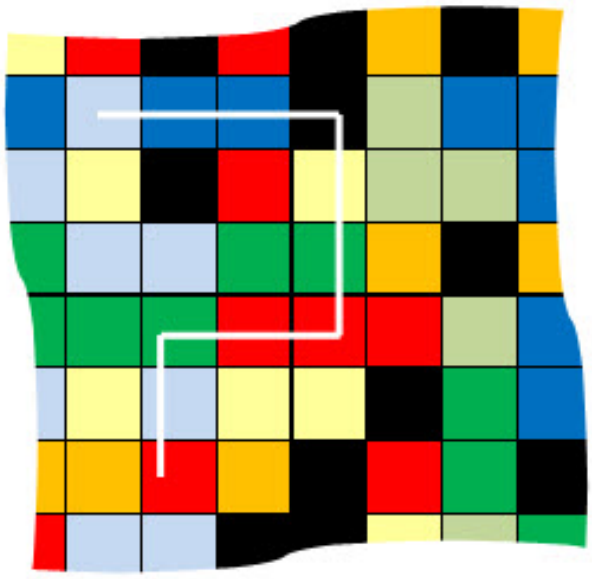

Figure 1: Any two squares can be connected by a rook's path

Where the color along the path changes, we get an edge in G. Thus, every two vertices of G are connected by a path along its edges, and hence, G is connected.

Any tree on 2011 vertices features the minimum number 2010 of edges in a connected graph (an easy proof by mathematical induction); the complete graph K_{2011} (i.e., the graph where every pair of vertices is adjacent) sports the maximum number $\binom{2011}{2}$ of edges. All there is left to demonstrate is a coloring of the grid that induces a tree graph T, and a coloring of the grid that induces the complete graph K_{2011}.

In order to achieve a tree, we first color the 2011×2011 square grid in a chessboard fashion in color 1 and a temporary color 2012. We then replace color 2012 with colors 2, 3, …, 2011 with each of these colors used on at least one unit square. We get a tree with the root at the vertex corresponding to color 1, and 2010 edges connecting this vertex to all other 2010 vertices of the graph (Fig. 2).

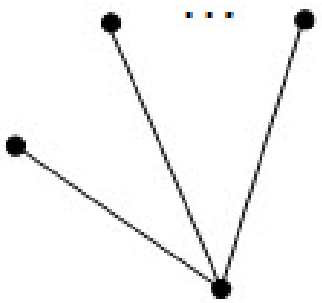

Figure 2: A tree induced by an appropriate grid coloring

Let us now induce the complete graph K_{2011}.

The row 1 we color in two alternating colors, 1 and the temporary color 2012. We then replace the color 2012 with one unit square of each of the colors 2, 3, …, 1006.

The row 2 we color in two alternating colors, 2 and the temporary color 2012. We then replace the color 2012 with one unit square of each of the colors 3, 4, …, 1007; and so on.

The row 2011 we color in two alternating colors, 2011 and the temporary color 2012. We then replace the color 2012 with one unit square of each of the colors 1, 2, …, 1005.

Due to a cyclic nature of coloring, it suffices to verify that color 1 neighbors on each of the other colors. It neighbors on colors 2, 3, …, 1006 in the first row. For $1007 \le i \le 2011$, color i neighbors on color 1 in the row i, where color i neighbors on colors i+1, i+2, … , i+1006 (we calculate these sums modulo 2011, i.e., subtract 2011 from the color number as soon as it exceeds 2011). Since $i + 1 \le 2012$ and $2013 \le i$+1006, color 1 will appear as a neighbor of color i in the row i. ■

This problem must have entered my subconscious universe – without me realizing it. Consequently I created the following related problem:

Problem of Neighboring Colors (A. Soifer). Each unit square of an $n \times n$ square grid is colored in one of m colors, so that every two colors somewhere on the grid are neighbors, i.e., share a side. Find maximum of m, which we will denote as max(m).

I immediately came up with colorings for small n, suggesting a conjecture:

Neighboring Colors Conjecture (A. Soifer). max(m) = $2n$–1.

It was not hard to show that for any n, max(m) $\le 2n$–1, but a proof of the conjecture in general case eluded me. And so I shared the problem with the 1990 and 1991 Colorado Mathematical Olympiad winner Matthew Kahle, now a Professor of Mathematics at Ohio State University, who in turn shared it with his Ph.D. student Francisco Martinez-Figueroa. The three of us achieved a good progress, but still did not prove the conjecture completely. First, we found that in 2010 Keith Edwards showed that max(m) = $2n$–1 for all sufficiently large n, but it was not clear how large n must be, and the proof was not constructive. We proved the following results constructively and published them in *Geombinatorics* during the pandemic in April-2020:

Small *n* Theorem [KMS]. For any $n \le 8$, max(m) = $2n$–1.

Lower Bounds Theorem [KMS]. For any positive integer n, $\max(m) \geq 2n-9$. Moreover:

*) if $n \equiv 0$ or $1 \pmod 4$, then $\max(m) \geq 2n-6$;
**) if $n \equiv 2 \pmod 4$, then $\max(m) \geq 2n-7$.

As you can see, a beautiful Olympiad problem led me to create an even more exciting problem, which proved to be too hard for the Olympiad. However, it gave three generations of mathematicians an opportunity to enjoy a joint work. It could give you a pleasure of advancing the proof (or disproof) of the conjecture further.

This has been a most depressing time in my 42 years on these shores. Yet as much as I could compose myself, I have been composing a new much expanded edition [4] of *The Mathematical Coloring Book*. Its ca. 600 pages promise to double. My white collie poppy Bellissimo takes me daily for a stroll around Quail Lake, and sometimes to high country for hunting. No, Bellissimo and I will never hurt our beautiful Colorado wild animals – we hunt for mushrooms. In the fall-2020 semester, I am teaching – in person – mathematics and my new course

"World War II History through Films of Individual Tragedy": https://www.youtube.com/watch?v=MZ_U9WYjwqQ&feature=youtu.be Join my course or just be my guests to view these incredible cinematic works of Polish, Russian, French, Lithuanian, and American film directors. As you can see from the photo above, taken on September 4, 2020, I am teaching in a helmet resembling a medieval knight's or a motorcyclist's protection. :)

Summing up, I made two observations while living with the epidemic:

> *Man has been trying to conquer Nature, and Nature responded with hurricanes, tsunami, and pandemics.*
>
> *Man created virtual reality as a reflection of his life, and virtual reality returned the favor.*

Bibliography

1. Soifer, A., Remembering John Horton Conway, *Geombinatorics* XXX(2), October 2020, 53–55.
2. Soifer, A., Remembering Ronald Lewis Graham, *Geombinatorics* XXX(2), October 2020, 56–62.
3. Kahle M., Martinez Figueroa, Francisco, and Soifer, A., A Square Grid Coloring Problem, *Geombinatorics* XXIX(4), April 2020, 167–184.
4. Soifer, A., *The Mathematical Coloring Book: Mathematics of Coloring nd the Colorful Life of Its Creators*, Springer, New York, in the works.